

Solutions to 2000 Applicable Mathematics Paper

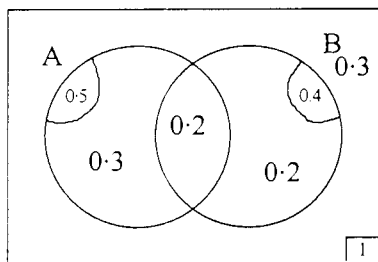
Question One

$$1 \quad (i) \quad A + 2B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} + 2 \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} = \underline{\underline{\begin{bmatrix} -1 & 6 \\ 3 & 12 \end{bmatrix}}}$$

$$(ii) \quad C + D = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ 3 & 2 & 4 \end{bmatrix} = \quad \text{NOT POSSIBLE. The matrices do not have the same size (order) which is necessary for addition.}$$

$$(iii) \quad A^{-1} = \text{NOT POSSIBLE. Since the determinant of } A = 0 \text{ i.e. } ad - bc = 0 \Rightarrow A \text{ is a singular matrix and hence } A^{-1} \text{ does not exist.}$$

Question Two



Solution one:

Using the diagram provided, $P(A \cap B) = 0.2$

$$P(A) = 0.5, P(B) = 0.4$$

$$P(A) \times P(B) = 0.5 \times 0.4 = 0.2$$

$$\text{since } P(A) \times P(B) = P(A \cap B)$$

Events A and B are independent

Alternative Solution:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5$$

$$P(A) = 0.5$$

$$\text{Since } P(A) = P(A|B)$$

Events A and B are independent

From the diagram:

Question Three

- (i) The new mean will represent a change of origin (All measures of central tendency will undergo a change in origin)
 \therefore New mean = (old data) + 3 = 7 + 3 = 10
 The old variance (measure of dispersion) will not undergo a change of origin
 \therefore New Variance = old variance = 5
- (ii) The new mean will represent a change in scale. (All measures of central tendency will undergo a change in scale)
 \therefore New mean = old data \times 2 = 7 \times 2 = 14
- (iii) The old variance (measure of dispersion) will undergo a change in scale of k^2
 \therefore New variance = (old data) \times k^2 = 5 \times 2² = 20

Question Four

- a) Let x = the length of tape the cassette holds

$$\therefore x \sim N(\mu = 185, \sigma^2 = 4)$$

$$\text{Hence } P(x > 180) = P(z > -2.5) = 0.0062096$$

$$\therefore \text{Percentage less than 3 hours} = \underline{0.62\% (2 \text{ dp})}$$

b) $P(x > 187) = P(z > 1) = 0.15866$

- c) Conditional probability based the movie being 3 hours and seven minutes long

$$\therefore P(x > 187 | x \geq 186) = \frac{P(x \geq 187)}{P(x \geq 186)}$$

$$= \frac{P(z \geq 1)}{P(z \geq 0.5)}$$

$$= \frac{0.15866}{0.30854}$$

$$\text{Probability Susie will record credits (as well)} = \underline{0.5142 (4\text{dp})}$$

Question Five

a) Let $\begin{bmatrix} 2 & 1 & 1 & 15 \\ 3 & 3 & 2 & 10 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 20 \end{bmatrix} = A$, $\begin{bmatrix} b \\ m \\ n \\ p \end{bmatrix} = X$ and $\begin{bmatrix} 21.45 \\ 21.50 \\ 3.90 \\ 26.70 \end{bmatrix} = B$

Hence from question: $AX = B$

PRE-multiplying both sides by A^{-1} yields

$$\therefore A^{-1} A X = A^{-1} B$$

$$\therefore I A X = A^{-1} B$$

$$\therefore \underline{X = A^{-1} B}$$

b) $A^{-1} = \begin{bmatrix} 4 & 0 & -1 & -3 \\ -2 & 1 & -1 & 1 \\ -2 & -1 & 3 & 2 \\ -0.2 & 0 & 0 & 0.2 \end{bmatrix}$ (obtained directly from G.C)

- c) Solving the system in Part a) using the inverse from Part B gives Bread = \$1.80 per loaf, Milk cost \$1.40 per litre, Newspapers cost 70c each and Petrol costs \$1.05 per litre.

Question Six

Rewriting the given set of equations in augmented matrix form gives

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 2 & 1 & -1 & 3 \\ 3 & -1 & 0 & a \end{array} \right]$$

The easiest way to proceed is to perform row reduction and then substitute the question values into the final matrix.

$$\text{i.e. } \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 5 & -3 & 1 \\ 0 & 5 & -3 & a-3 \end{array} \right] \begin{array}{l} R1 \\ R2 \rightarrow R2 - 2R1 \\ R3 \rightarrow R3 - 3R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 5 & -3 & 1 \\ 0 & 0 & 0 & a-4 \end{array} \right] \begin{array}{l} R1 \\ R2 \\ R3 \rightarrow R3 - R2 \end{array}$$

When $a = 4$

The last line of matrix becomes $[0 \ 0 \ 0 \ | \ 0]$

\Rightarrow An INFINITE NUMBER OF SOLUTIONS are possible.

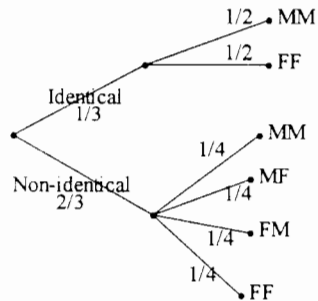
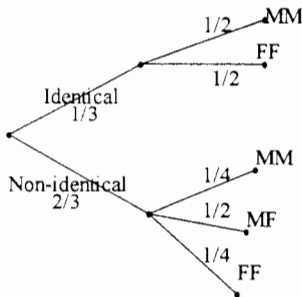
When $a = 5$

The last line of the matrix becomes $[0 \ 0 \ 0 \ | \ 1]$

\Rightarrow NO SOLUTION is possible.

Question Seven

From the question it can be assumed that twins will be either identical OR non-identical. Furthermore, identical twins would have to have the same gender, whereas non-identical twins could be mixed genders. Possible tree diagrams include:



$$(i) \quad P(\text{Two Boys}) = P(I, B) + P(NI, B) \\ = \left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right)$$

$$(ii) \quad P(\text{Two children same sex}) = \frac{1}{3} \\ = P(GG) + P(BB) \\ = P(I, G) + P(NI, G) + P(I, B) + P(NI, B) \\ = \left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right) + \left(\frac{1}{3}\right) \\ \text{answer from part a)} \\ = \underline{\underline{\left(\frac{2}{3}\right)}}$$

$$(iii) \quad P(I|GG) = \frac{P(I \cap GG)}{P(GG)} = \frac{\frac{1}{3} \times \frac{1}{2}}{\left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right)} \\ = \underline{\underline{\left(\frac{1}{2}\right)}}$$

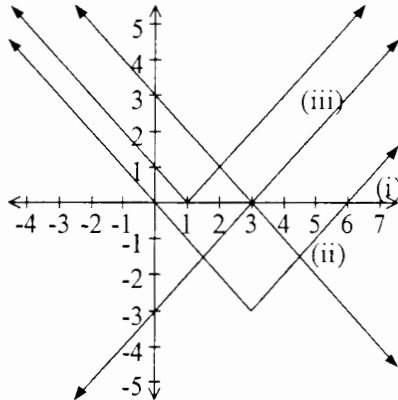
$$(iv) \quad P(I|\text{At least one girl}) = \frac{P(I \cap \text{At least one girl})}{P(\text{At least one girl})} \\ = \frac{\frac{1}{3} \times \frac{1}{2}}{\left(\frac{1}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{1}{4}\right)}$$

Question Eight

- a) The length of the cycle of the graph is 12 months or 1 year
- b) A slight positive value for the gradient of the trend line tends to indicate a slightly increasing trend.
- c) Using the gradient of the trend line which represents the average rate of change
 The average annual increase = 0.0223×12 million
 = 0.2676 million
 or 267 600
- d) A = $7.360 + 0.0223(8) = \underline{7.54}$
 B = 9.3
- e) The seasonal component = average residual for December = $\frac{0.97 + 0.90 + 1.14}{3} = \underline{1.003}$
- f) December 2000 corresponds to $t = 48$ in data
 $\therefore N(48) = 7.360 + 0.0223(48) = 8.4304$
 \therefore Prediction = trend + seasonal component
 = $8.4304 + 1.0033$
 = 9.434 million

Question Nine

- (i) $y = f(x - 2) - 3$ represents a vertical translation down of 3 units
(ii) $y = -f(x - 2)$ represents a reflection in the x-axis
(iii) $y = f(x)$ will represent a horizontal translation 2 units to the left



Question Ten

It is important for students to realise that the information in the table is detailing the DURATION of a certain number of calls. It is not a table of frequency / group data.

- a) i) Average Duration = $\frac{\text{total duration of calls}}{\text{no. of calls}} = \frac{1175}{283} = \underline{\underline{4.152 \text{ mins}}}$
ii) Standard Deviation for the number of calls per month = 18.197 (Calculator)
- b) Average Telephone Bill per month = connection fee + $\frac{\text{total duration of calls}}{6} \times 0.05$
= $\$5 + \frac{1175}{6} \times 0.005$
= $\underline{\underline{\$14.79}}$
- c) Average + 2 Standard deviations = $47.17 + 2 (18.197) = 83.564$
Hence MONTH 2 with 86 calls has an unusually large number of calls per month.

Question Eleven

$$n = 141$$

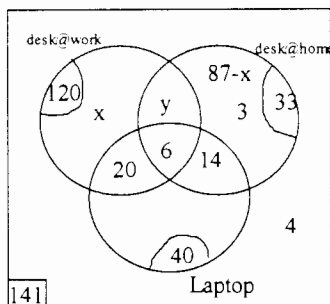
$$n(\text{desk@w}) = 120$$

$$n(\text{Lap}) = 40$$

$$n(\text{Desk@H}) = 33$$

Let $x = N^{\circ}$ people with desktop computer only

$y = N^{\circ}$ of people with a desktop at work and at home but no laptop



$$\begin{aligned} x + y &= 94 \\ 87 - x + y &= 13 \\ \Rightarrow x &= 84, \quad y = 10 \end{aligned}$$

Hence N° people with no computer = 4

Question 12

It is always challenging assigning a $t = 0$ value with time series data presented in years. Students would be expected to make a decision on which year $t = 0$ is occurring from the information presented and state this clearly.

a) $r_{sk} = 0.9733$

b) If $t = 0$ is @ 0 $\Rightarrow K(t) = -4585.29 + 2.3619t$

If $t = 0$ is @ 1900 $\Rightarrow K(t) = -97.5975 + 2.3619t$

If $t = 0$ is @ 1979 $\Rightarrow K(t) = 88.997 + 2.3619t$

(There seems many possibilities here, but those above would be the most reasonable)

c) (i) Year 2005 $\Rightarrow K(2005) = 150.3$ million } Using each
 $\Rightarrow K(105) = 150.4$ million } model above.
 $\Rightarrow K(26) = 149.3$ million }

A range of answers would be acceptable here.

(ii) It is not very reliable because Year 2005 can be seen to be an extrapolation outside the data range of the given data.

d) The average number of kilometres will decrease with time so expect the correlation coefficient to be negative. However the total number of kilometres travelled is increasing with time, so the correlation coefficient would be positive \therefore Hence OPPOSITE SIGNS OBSERVED. This would suggest that the number of vehicles is increasing.

Question Thirteen

$$a) \begin{matrix} P_1 & Q_1 & R_1 & P_2 & Q_2 & R_2 \\ \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} & = & \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix} & \begin{bmatrix} 0 \\ 2 \end{bmatrix} \end{matrix}$$

Coordinates are $P_2(-1,-2)$, $Q_2(0,-2)$, $R_2(0,2)$

$$b) \begin{matrix} P_2 & Q_2 & R_2 & P_3 & Q_3 & R_3 \\ \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix} & \begin{bmatrix} 0 \\ 2 \end{bmatrix} & = & \begin{bmatrix} -4 & -2 \\ -2 & -2 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \end{bmatrix} \end{matrix}$$

Coordinates are $P_3(-4,-2)$, $Q_3(-2,-2)$, and $R_3(2,2)$

c) Solution one.

Let the transformation matrix be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then we require

$$\begin{matrix} P_3 & R_1 \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & \begin{bmatrix} -4 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{matrix} \Rightarrow \begin{matrix} -4a - 2b = 1 \\ -4c - 2d = 2 \end{matrix}$$

$$\begin{matrix} Q_3 & Q_1 \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \end{matrix} \Rightarrow \begin{matrix} -2a - 2b = 0 \\ -2c - 2d = 2 \end{matrix}$$

$$\begin{matrix} R_1 \\ \begin{bmatrix} a & b \\ c & d \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \end{matrix} \Rightarrow \begin{matrix} 2a + 2b = 0 \\ 2c + 2d = -2 \end{matrix}$$

solving simultaneously

$$a = -\frac{1}{2}, \quad b = \frac{1}{2}, \quad c = 0, \quad d = -1$$

$$\therefore \text{Required Matrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 0 & -1 \end{bmatrix}$$

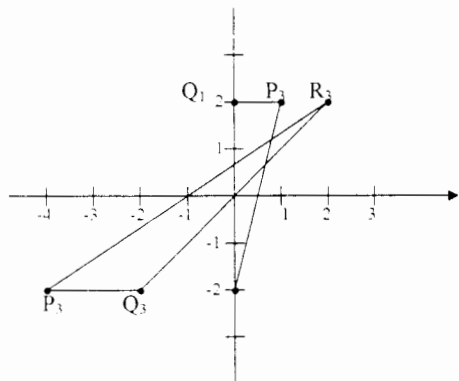
Alternative Solution

$$\left[\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right]^{-1} = \begin{bmatrix} -2 & -1 \\ 0 & -1 \end{bmatrix}^{-1}$$

$$= \frac{1}{2-0} \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 0 & -1 \end{bmatrix}$$

d)



$$\text{Area } \Delta P_1 Q_1 R_1 = \frac{1}{2} \cdot 1 \cdot 4 = 2$$

$$\text{Area } \Delta P_3 Q_3 R_3 = \frac{1}{2} \cdot 2 \cdot 4 = 4$$

Ratio Area Δ_3 to Area Δ_1

Alternatively :

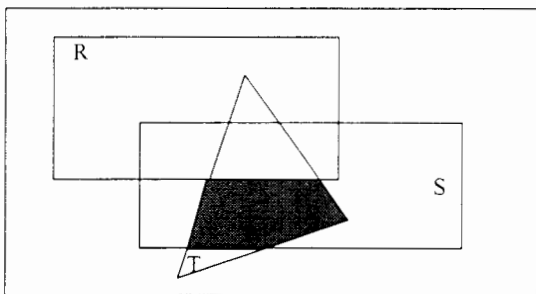
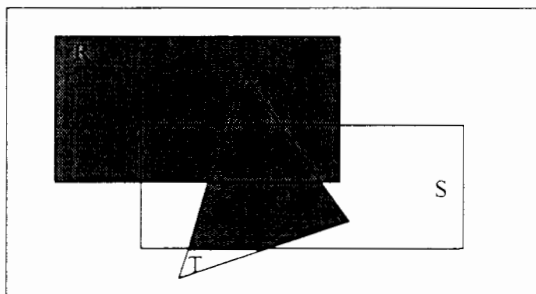
$$\det \text{ of } \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = 1$$

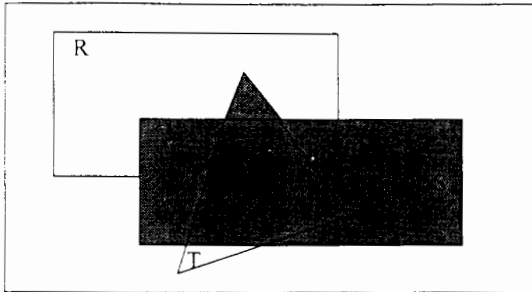
$$\det \text{ of } \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 0 & -1 \end{bmatrix} = 1$$

\therefore Ratio is $2 : 1$
or $1 : 0.5$

Question 14

a)





Question Fifteen

$$a) \quad AC = \begin{bmatrix} 3 & 3 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 2 & 2 \end{bmatrix}$$

$$BC = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 9 \\ 2 & 2 \end{bmatrix}$$

- b) To answer the question, students must find a condition that will result in the statement being true

$$\begin{array}{lcl} \text{i.e.} & \text{If} & DX = EX \\ & \text{then} & DX X^{-1} = EX X^{-1} \\ & \text{i.e.} & DI = EI \\ & & D = E \end{array}$$

Hence the statement is true providing matrix X is NOT singular, that is, it has an inverse.

Question Sixteen

- a) The EXPONENTIAL MODEL is more appropriate for the data because there is more of a random pattern with the residual plot (The residual plot for the linear model appears to be following a quadratic pattern)
- b) It is always challenging assigning a $t = 0$ value with time series data presented in years. Students would be expected to make a decision on which year $t = 0$ is occurring from the information presented and state this clearly.

$$\text{If } t = 0 @ 0 \text{ was chosen then } y = 1.6602 \times 10^{-19} e^{0.02545t}$$

$$\text{or } y = \frac{1.6602 \times 10^{-19} \times 1.02577^t}{}$$

$$t = 0 @ 1800 \text{ was chosen then } y = 12.9835 e^{0.02545t}$$

$$\text{or } y = \frac{12.9835 \times 1.02577^t}{}$$

$$t = 0 @ 1896 \text{ was chosen then } y = 149 \cdot 40e^{0.02545t}$$

$$\text{or } y = \frac{149 \cdot 40 \times 1.0577^t}{}$$

There seems many possible answers, but these are the most likely acceptable choices.

$$\begin{aligned} \text{c) } \% \text{ over } 65 &= \frac{(1996)\text{value}}{20.8 \text{ million}} \times 1000 \\ &\approx \underline{\underline{0.091 \text{ or } 9.1\%}} \end{aligned}$$

Question Seventeen

a) Let $X = \#$ of Paying Customers per hour

$$\therefore N^{\circ} \text{ average number of paying customers} = \frac{60}{40} = \underline{\underline{1.5}}$$

b) Let $X = \#$ Arriving

$$\therefore X \sim \text{Poisson } (\mu = 1.5)$$

$$\therefore P(X = 2) = \underline{\underline{0.25102}}$$

$$\begin{aligned} \text{c) } P(X > 2) &= 1 - P(X \leq 2) \\ &= 1 - 0.80884 \\ &= \underline{\underline{0.19116}} \end{aligned}$$

d) NQ. He should not employ extra help in this instance since 0.1916 (19%) is < 30%

e) Average time in minutes between the arrival of two paying customers is $\frac{40}{60} \times 60 = \underline{\underline{40 \text{ mins}}}$

f) Let $Y =$ the time for the next customer to arrive

$$\therefore Y \sim e \left(\lambda = \frac{1}{40} \right) \quad [\text{i.e. } f(t) = \frac{1}{40} e^{-\frac{1}{40}t}]$$

$$\begin{aligned} \therefore P(Y \geq 20) &= e^{-\frac{20}{40}} - e^{-\infty/40} \\ &= e^{-0.5} \\ &= \underline{\underline{0.60653}} \end{aligned}$$

g) $P(\text{At least one}) = 1 - P(\text{none})$

$$P(\text{None}) = P(Y \geq 90 | Y \geq 60) = P(Y \geq 90) / P(Y \geq 60)$$

$$= P(Y \geq 30) = e^{-\frac{30}{40}} = 0.47237$$

$$\therefore P(\text{at least one}) = \underline{\underline{0.52763}}$$

Question Eighteen

- a) Students need to realise that summarised data presented is in groups. Hence MIDPOINTS of the mark range will have to be used.

(The midpoints are 9.5, 29.5, 49.5, 69.5, 89.5)

The calculator is able to give the values directly for $\bar{x} = 62.83$ and Standard Deviation = 22.54

- b) (i) $P(X \geq 60) = \frac{32}{48} = \frac{2}{3}$ (directly from table)
- (ii) $P(X = \text{Applic}) = \frac{27}{48} = \frac{9}{16}$ (directly from table)
- (iii) $P(\text{Calc} \mid X \geq 80) = \frac{5}{48}$
- (iv) $P(\text{Calc} \mid 60 \leq X \leq 79) = \frac{9}{20}$

- c) Solution one

$$\begin{aligned}P(\text{at least 1}) &= 1 - P(\text{none have marks} \geq 80) \\&= 1 - \left(\frac{36}{48} \times \frac{35}{47} \times \frac{34}{46} \times \frac{33}{45} \right) \\&= \underline{0.69727}\end{aligned}$$

Alternative Solution (using a Counting Technique approach)

$$\begin{aligned}P(\text{at least 1}) &= 1 - \frac{\binom{36}{4}}{\binom{48}{4}} \\&= \underline{0.69727}\end{aligned}$$

- d) Let X = the number of students walking to school
 $X \sim b(n = 6, p = 0.25)$

$$\begin{aligned}\therefore P(X \geq 3) &= 1 - P(X \leq 2) \\&= 1 - 0.83056 \\&= \underline{0.16944}\end{aligned}$$

Question Nineteen

It is very important that students define their variables in matrix problems involving systems of equations to be solved. This question specifically asked for it too!

∴ Let n , e , s and w be the number of vehicles leaving the roundabout by the North, East, South and West roads respectively.

$$\begin{array}{l} n + e + s + w = 6000 \\ \text{i.e. } s = w(n + e) \\ e = \frac{w}{3} \\ n = c + 500 \end{array} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{From the information supplied} \\ \text{on the question} \end{array}$$

These equations simplify to

$$\begin{array}{rccccrcl} n & + & e & + & s & + & w & = & 6000 \\ 2n & + & 2e & - & s & & & = & 0 \\ & & 3e & & & - & w & = & 0 \\ n & - & e & & & & & = & 500 \end{array}$$

Hence in matrix form (and not in augmented form)

Since the question did insist on NOT using this form

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & -1 & 0 \\ 0 & 3 & 0 & -1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} n \\ e \\ s \\ w \end{bmatrix} = \begin{bmatrix} 6000 \\ 0 \\ 0 \\ 500 \end{bmatrix}$$

Question Twenty

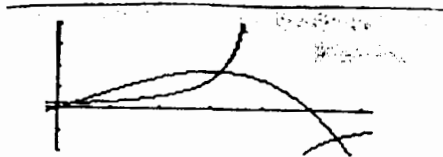
When working with a question of this nature (one that requires a graphics calculator solution), students must relate the equations provided to the CONTEXT outlined by the equation.

The context is modelled by a function that describes how the situation is changing. Hence the interpretation of the graph of that function becomes vital.

Note: This question is best not to be attempted algebraically!!!

Method:

1. Draw the function on the calculator and locate the point of intersection in the domain $0 \leq t \leq 3.5$



2. Observe where $k(t) > c(t)$ and interpret

i.e. Chris has a lower score between 0.30 mins and 3.19 mins

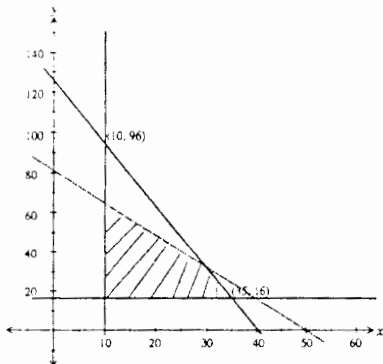
Question Twenty One

From the question, the following inequalities hold

a) $3 \cdot 2x + y \leq 128$ }
 $x \geq 10$ } These are already drawn.
 $y \geq 16$ }

Time to be added is $0 \cdot 8x + 0 \cdot 5y \leq 40$

The shaded region has been identified



b) Profit = $100x + 40y$

Points	Profit
(10, 16)	\$1640
(35, 16)	\$4140
(30, 32)	\$4280
(10, 64)	\$3560

Need to manufacture 30 jackets and 32 vests for \$4280 profit.

c) If the new profit equation is $ax + 40y$ (where $a > 100$), then the slope of this line will be $-\frac{a}{40}$
 For greatest flexibility the profit line should coincide with one of the lines through (30, 32).

The slope of the two lines involved is $-3 \cdot 2$ and $-1 \cdot 6$.

Hence $-\frac{a}{40} = -3 \cdot 2$ and $-\frac{a}{40} = -1 \cdot 6$

i.e. $a = \$128$ and $a = \$64$

a should be 128 \Rightarrow an increase of \$28.

Points on the line (30, 32) and (35, 16) satisfy.
 Hence should manufacture 30 jackets and 32 vests OR 35 jackets and 16 vests.

Question Twenty Two

It would be assumed that the students would recognise that the integration of the functions provided is far too complicated algebraically and students would use a graphics calculator directly. However, writing the actual integration statement would be of extreme value to a student who had pressed an incorrect key and was unaware of the error.

$$\text{a) } \int_1^{15} (t-1)^2 e^{-(t-1)} dt = \underline{0.99991}$$

$$\text{b) } \int_7^{27} (t-1)^2 e^{-(t-1)} dt = \underline{0.06197}$$

$$\text{c) } \int_{2.6}^9 (t-1)^2 e^{-(t-1)} dt = \underline{0.76960}$$

Question Twenty Three

- a) Let $X \neq Y$ be the number having the disease

$$X \sim \text{binomial} \quad \text{i.e.} \quad X (n = 900, p = 0.01)$$

$$Y \sim \text{normal} \quad \text{i.e.} \quad Y (\mu = 9, \sigma = 2.98496)$$

$$P(8 \leq X \leq 12) \approx P(7.5 \leq Y \leq 12.5) \\ = \underline{0.57185}$$

- b) Let Z = the number with disease

$$Z \sim \text{Poisson i.e.} \quad Z \sim P(\mu = 9)$$

$$\therefore P(8 \leq X \leq 12) \approx P(8 \leq Y \leq 12) \\ = P(Y \leq 12) - P(Y \leq 7) \\ = 0.857577 - 0.32389 \\ = \underline{0.55188}$$

- c) Since 0.55188 is closer to 0.55415 than 0.57185, there is evidence to support the hypothesis given in the question.

i.e. The Poisson distribution is better approximation than the normal to the binomial in this situation.

A note to good students:

There are many instances in this paper where an answer can be obtained directly from the graphics calculator and it seems cumbersome to show working as demonstrated by these solutions. However, students must realise that in an examination they are required to **demonstrate their understanding** and often a response from a graphics calculator may be incorrect due to a syntax error or input error. If the student is unaware of the "incorrectness" of the screen display, this can lead to the student believing they have completed the question correctly when in actual fact this is not the case. Such a situation would receive little, (or possibly zero) marks for the question as a stated incorrect answer without any working often receives low marks. Good students consider providing support for all the responses.

Acknowledgement –

**Many thanks to
David Enright**